Kriging-based surrogate model for the solution of inverse problems in nondestructive testing

Sándor Bilicz*†, Marc Lambert†, Szabolcs Gyimóthy* and József Pávó*
*Budapest University of Technology and Economics, Egry J. u. 18, H-1521 Budapest, Hungary
†Département de Recherche en Électromagnétisme,
Laboratoire des Signaux et Systèmes UMR8506 (CNRS-SUPELEC-Univ Paris-Sud), 91192 Gif-sur-Yvette cedex, France
Email: bilicz@evt.bme.hu

***Abstract*—The inverse problems of electromagnetic nondestructive testing are often solved via the solution of several forward problems. For the latter, precise numerical simulators are available in most of the cases, but the associated computational cost is usually high. Surrogate models –getting more and more widespread in electromagnetics– might be promising alternatives of heavy simulations. Traditionally, such surrogates are used to replace the forward model. However, in this paper the direct use of surrogate models for the solution of an inverse problem is studied and illustrated by eddy-current testing examples.**

***Index Terms*—Surrogate modeling; Kriging; Inverse problem; Nondestructive evaluation; Eddy-current testing**

I. Introduction

The electromagnetic nondestructive evaluation (ENDE) –the characterization of in-material defects based on the measured electromagnetic field– still remains a challenging issue. Industrial applications need fast and reliable inversion methods, whereas the numerical simulators of the involved electromagnetic (EM) phenomena are computationally expensive.

Surrogate models are getting more and more widespread alternatives of EM simulators [1]. Kriging –a stochastic function approximation technique (see, e.g. the recent overview of [2])– is often used in such surrogate models. It has been applied as an approximated forward simulator with success for EM device optimization (e.g., [3], [4]) and the inverse problem has also been addressed [5], [6]. However, inversion can directly be performed –i.e., considering the measured data as input and the defect description is the output– via surrogate models, as proposed by, e.g., [7] using a neural network.

In this paper, a kriging-based surrogate modeling approach is proposed for such direct solution of ENDE inverse problems. The kriging model is based on some pre-computed (simulated) results, which are chosen by an adaptive strategy in order to improve the performance of the yielded surrogate model.

II. The proposed surrogate modeling approach

A. Inverse interpolation using a pre-computed database

Let us imagine an ENDE setup in which a defect is described by a finite number of real parameters –collected into the *input* vector $\mathbf{x}$. The set of all conceivable defects spans the *input space* $\mathbb{X}$. One can observe the functional output data $\mathcal{Z}(t)$. In eddy-current testing (ECT), the $\mathbf{x}$ input typically consists in the geometrical parameters of the defect, $\mathcal{Z}(t)$ is the impedance variation of the ECT probe at the position t, respectively. A numerical simulator for the studied configuration is also assumed to be at hand, which can compute the approximate output signal $Z(t)$ for an arbitrary defect given by the input $\mathbf{x}$.

The inverse problem roughly consists in determining $\mathbf{x}$ based on the knowledge of $\mathcal{Z}(t)$. For this purpose, we propose a kriging interpolation scheme. Let us assume that n simulations have been carried out, thus, n "samples" (corresponding input-output data pairs) are stored in a database:

$$\mathbb{D}_n = \{(\mathbf{x}_1, Z_1(t))\,, (\mathbf{x}_2, Z_2(t))\,, \ldots, (\mathbf{x}_n, Z_n(t))\}\,. \tag{1}$$

The interpolation can then be written as:

$$\widehat{\mathbf{x}} \approx \mathcal{I}_n\{\mathcal{Z}(t)\} \equiv \sum_{i=1}^{n} \lambda_i^I\left(\|\mathcal{Z}(t) - Z_i(t)\|\right)\mathbf{x}_i, \tag{2}$$

where $\widehat{\mathbf{x}}$ is the approximate solution of the inverse problem if the observation is $\mathcal{Z}(t)$, $\mathcal{I}_n$ denotes the "inverse operator" based on the n samples. The coefficients λ_i^I are computed in the stochastic framework of kriging (see, e.g., [2], [8]).

B. Adaptive generation of the database

The following adaptive sequential sampling scheme is proposed to build $\mathbb{D}_n$:

1) Choose a small n number of "initial input samples" in $\mathbb{X}$ by a classical space-filling design (e.g., full factorial [9]) and compute the corresponding outputs by the simulator at hand.
2) The next, $(n+1)$th sample is inserted so that the precision of (2) improves, i.e., the discrepancy between the real and predicted inputs ($\mathbf{x}$ and $\widehat{\mathbf{x}}$) decreases as much as possible, all over $\mathbb{X}$. To this end, the next sample is added where this discrepancy is presumably maximal:

$$\mathbf{x}_{n+1} = \arg\max_{\mathbf{x}\in\mathbb{X}} \|\mathbf{x} - \mathcal{I}_n\{Z(t)\}\|\,, \tag{3}$$

where $Z(t)$ is the output signal corresponding to $\mathbf{x}$. Since this optimization problem is computationally expensive due to the need for $Z(t)$, the latter is also approximated by kriging:

$$\tilde{Z}(t) = \sum_{i=1}^{n} \lambda_i^F\left(\|\mathbf{x} - \mathbf{x}_i\|\right) Z_i(t) \tag{4}$$

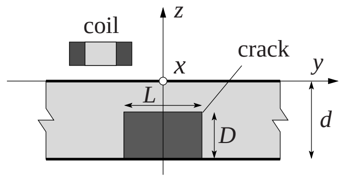

Figure 1: Cross-section of the studied ECT configuration. The depth D is given in percentages of the plate thickness d.

is used in (3) instead of $Z(t)$. The coefficients λ_i^F are computed similarly to λ_i^I.

3) $Z_{n+1}(t)$ is computed and $(\mathbf{x}_{n+1}, Z_{n+1}(t))$ is inserted to $\mathbb{D}_n$. Increase $n := n + 1$ and go to step 2 until a stopping criterion (e.g., upper limit for n) is met.

This scheme –using two successive kriging predictions per iteration– yields a database $\mathbb{D}_n$ which is adapted to the studied problem and to the applied interpolator (2).

The generation of the database might be time-consuming due to the forward simulations and the kriging operations (estimation of model parameters), but this task is performed only once. Then, the use of the surrogate model (2) is very fast (the coefficients λ_i^I $(i = 1, 2, \ldots, n)$ are computed via the solution of the system of $n + 1$ linear equations).

III. Performance on a simple ECT example

The approach is illustrated by a simple eddy-current testing (ECT) example. A homogeneous, non-magnetic conductive plate is affected by a thin, rectangular-shaped crack. An air-cored pancake type coil driven by time-harmonic current scans above the plate (Fig. 1). The variation of the coil impedance $\mathcal{Z}(t)$ is measured (t is the coil position over a rectangular surface). The position and the orientation of the crack are known, only its length L and depth D are enabled to vary. For the numerical simulation, an integral formalism [10] is used. The *input* is then $\mathbf{x} = [L, D]$, the input space is defined as $1\,\text{mm} < L < 10\,\text{mm}$ and $5\,\% < D < 90\,\%$. The optimization task (3) is solved via an exhaustive search, using a regular 37×18 grid on the (L, D) plane. The quality of the interpolation (2) is evaluated via the interpolation error:

$$\varepsilon(\mathbf{x}) = \sqrt{\left[\left(\widehat{L} - L\right)/L\right]^2 + \left[\left(\widehat{D} - D\right)/D\right]^2}, \qquad (5)$$

where $[\widehat{L}, \widehat{D}]$ are given by the interpolator (2), assuming that $\mathcal{Z}(t) = Z(t)$, corresponding to $[L, D]$. A comparison of a classical and the adaptive sampling (Fig. 2) shows a better performance of the latter with respect to the former.

IV. Conclusion

The proposed kriging-based surrogate modeling approach shows good performance in the fast solution of inverse problems of the preliminary ECT examples. The future research will consider the effect of the noise on the observations. Further numerical studies (preliminary results have been obtained using 4 parameters) will be presented in an extended paper.

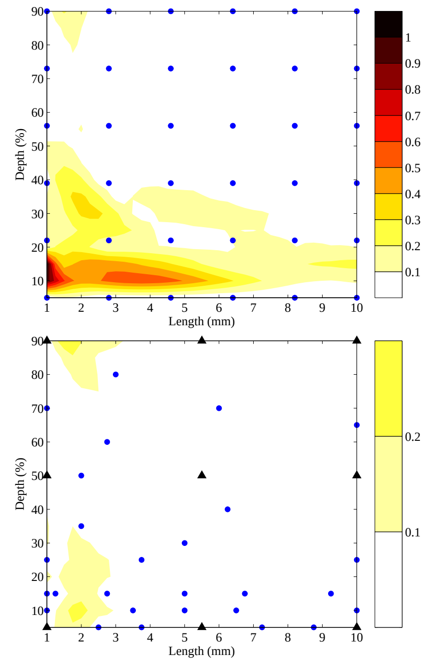

Figure 2: Interpolation error $\varepsilon(\mathbf{x})$ (colormap, evaluated in a 37×18 grid) and input samples $\mathbf{x}_i$. Top: classical sampling with 36 samples. Bottom: adaptive sampling: 9 initial (triangles) + 27 sequentially added samples.

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